GNSS signal raytracing

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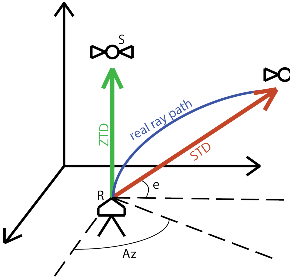
Outline

GNSS Slant Delay

Total delay along the ray path:

$$\Delta\tau = \int n(s)ds - S = \underbrace{\int (n(s)-1)ds}_{slowing\ \ effect} + \underbrace{\left(\int ds - S\right)}_{bending}$$

where s is path length, n is refractive index and S is straight-line length.



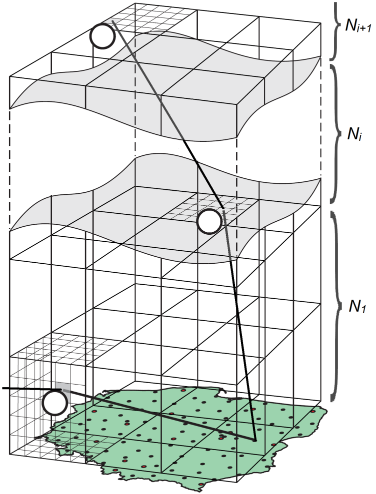
Slant Total Delay:

$$STD = SHD + SWD = \\ = m_h(e)ZHD + m_w(e)ZWD$$

Elements:

- R-S between receiver and satellite
- Az at given azimuth
- e at given elevation angle

Raytracing theory



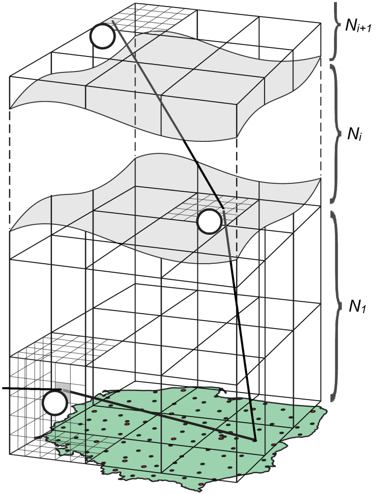
Mathematical model

- ray path length
- coordinates along the ray path
- signal bending in a function of refractive index

Meteorological data

- radiosonde profiles
- standard atmosphere
- Numerical Weather Prediction

Raytracing theory



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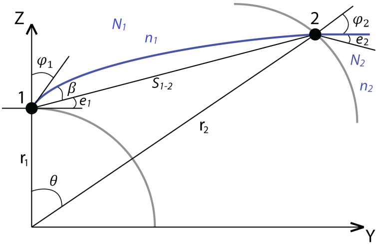
Mathematical model

Eikonal equation for geometric optics approximation (Born and Wolf, 1999):

$$\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right) = \nabla n$$

- $\vec{r}$ is a position vector
- s is a ray trajectory length
- n is a refractive index
- ∇n is a refractive gradient



Refractive index: $n = 1 + N \cdot 10^{-6}$

- N is a total refractivity

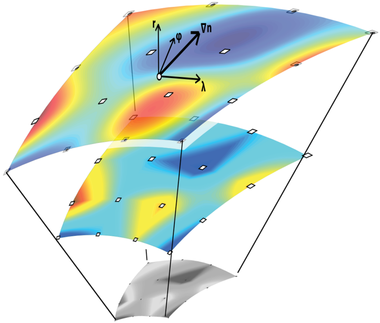
Eikonal solver (3D)

- the exact solution

$$\frac{d\vec{r}}{ds} = \frac{\vec{v}}{n(\vec{r})} \qquad \frac{d\vec{v}}{ds} = \nabla n(\vec{r})$$

- time-demanding processing
- requires small vertical spacing (1 m)
- calculation of refractivity and partial derivatives at each iteration:

$$\frac{\partial n(r,\lambda,\phi)}{\partial r}, \quad \frac{\partial n(r,\lambda,\phi)}{\partial \phi}, \quad \frac{\partial n(r,\lambda,\phi)}{\partial \lambda}$$



where $\vec{v}$ is tangent vector of trajectory at point $\vec{r}$, (r, λ, ϕ) are spherical coordinates

Thayer approach (2D)

- approximate solution

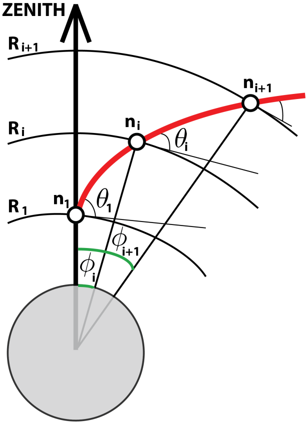
$$\frac{n}{n_1} = \left(\frac{r}{R_1}\right)^A$$

- faster ray-tracing computation
- horizontally stratified atmosphere

$$\frac{\partial n(r,\lambda,\phi)}{\partial \phi} = 0, \quad \frac{\partial n(r,\lambda,\phi)}{\partial \lambda} = 0$$

- designed for non-zenith delays in azimuthal symmetry
- signal delay:

$$\tau_{i+1} = \frac{R_i n_i cos\theta_i}{1 + A_{i+1}} \left(tan\theta_{i+1} - tan\theta_i\right)$$



Linear approach (2D)

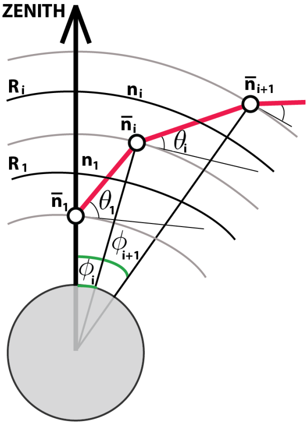
- based on Snell's law

$$n_i r_i sin\theta_i = n_{i+1} r_{i+1} sin\theta_{i+1}$$

- linear ray-path at constant azimuth
- fast calculation
- mean refractivity at half-heights
- signal delay:

$$\tau_{i+1} = \frac{\bar{n}_{i+1} - \bar{n}_i}{log\left(\frac{\bar{n}_{i+1}-1}{\bar{n}_i-1}\right)}\Delta s$$

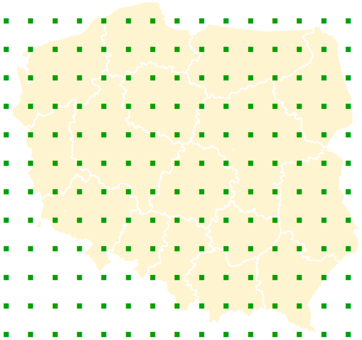
where Δs is geometrical distance between two points



Meteorological data

Weather Research and Forecasting model (WRF)

- real and idealized data
- staggering Arakawa-C grid
- vertical grid spacing: altitude dependent
- upper limit $\approx 30\ km$
- 24 hours forecast
- runs at 0.00 UTC



Horizontal resolution:

- country-wide: 13 km
- local: 4 km

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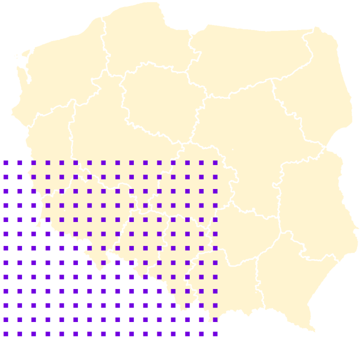

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Reference system

METEOROLOGICAL DATA IN GEODETIC REFERENCE SYSTEM

COORDINATE SYSTEMS

- NWP in spherical coordinates (geocentric)
- processing in geodetic coordinates

HEIGHT SYSTEMS CONVERSION

- meteorological parameters at geopotential levels
- raytracing in geometric altitudes

$$h = \frac{\zeta \cdot g_0}{\overline{\gamma}(\phi, \lambda, H)} + N$$

EARTH RADIUS

- Gaussian

$$R = \frac{a^2 b}{(acos\phi)^2 + (bsin\phi)^2}$$

- Euler formula

$$R = \left(\frac{cos^2 a}{M} + \frac{sin^2 a}{N} \right)^{-1}$$

- osculating sphere

$$R = \frac{a\sqrt{1-e}}{1-e^2 sin^2 \phi}$$

where $\overline{\gamma}(\phi, \lambda, H)$ is mean acceleration due to gravity g_0, N is geodetic undulation, ζ is geopotential height, e is ellipsoid eccentricity at semi-major axis a and semi-minor axis b

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Input parameters

Air pressure P

⇓

refractivity coeffcient k_2'

$$k_2' = k_2 - k_1 \frac{M_w}{M_d}$$

Air temperature T

⇓

degrees conversion

Celsius [^oC] $\Longrightarrow$ Kelvin [K]

Relative humidity RH
Specific humidity Q

⇓

water vapor pressure

$$e = e_s \frac{RH}{100}$$

$$e = \frac{Q \cdot P}{0.622 + 0.378Q}$$

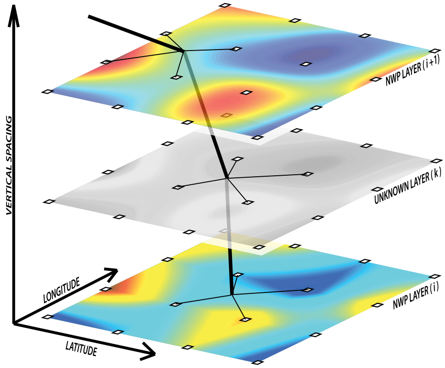
⇓

refractivity (refractive index)

$$N = k_1 \frac{P}{T} + k_2' \frac{e}{T} + k_3 \frac{e}{T^2} \Longrightarrow n = 1 + N \cdot 10^{-6}$$

where M_w and M_d are molar masses of wet and dry air, respectively 18.0152 and 28.9644 g/mol

Interpolation in NWP model



1. limited vertical resolution
   - integration step (iterations)
   - interpolation between vertical layers
   - supplementary atmosphere over upper limit
2. interpolation at horizontal layer
3. time interpolation of parameters
   - for observation epoch
   - 3h/6h/12h forecast runs

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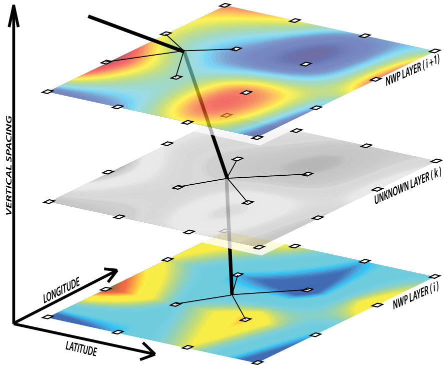

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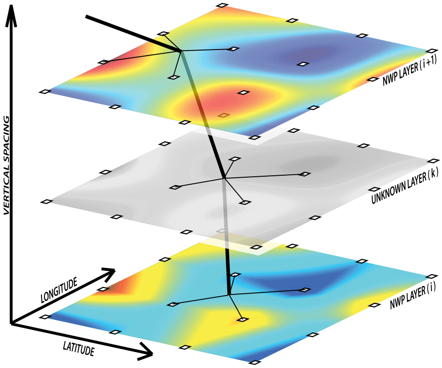

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Horizontal / vertical interpolation

HORIZONTAL

- NEAREST

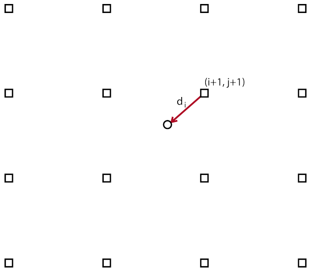
  $P = P(d_i)$, where $d_i = min$

- WEIGHTED AVERAGE

$$P = \frac{\sum_{i=1}^{n} P_i w_i}{\sum_{i=1}^{n} w_i}, \text{ where } w_i = [(x - x_i)^2 + (y - y_i)^2]^{-2}$$

- SPLINE

$$P = \sum_{i=1}^{4} \sum_{j=1}^{4} a_{ij} x_i y_j$$



d_T is temperature lapse rate, g is gravity acceleration, M_d is molar mass of dry air, R is universal gas constant

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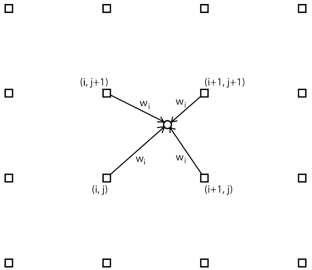

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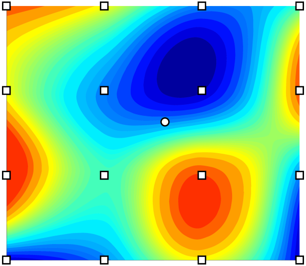

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VERTICAL

- LINEAR

$$T = T_i - d_T(h_i - h)$$

- EXPONENTIAL

$$P = P_i \left(\frac{T_i + d_T(h_i - h)}{T_i} \right)^{\frac{g \cdot M_d}{R \cdot d_T}}$$

- LOGARITHM

$$logP = logP_i + \frac{h_i - h}{18400 \left(1 + \frac{T + T_i}{546}\right)}$$

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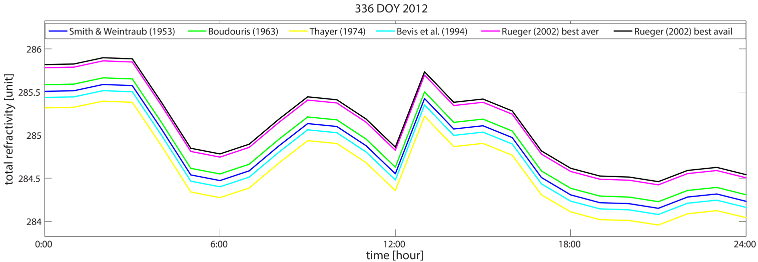
Refractivity coefficients



$$N = (n-1)\cdot 10^6 = k_1 \frac{P}{T} + k_2' \frac{e}{T} + k_3 \frac{e}{T^2} \qquad k_2' = k_2 - k_1 \frac{M_w}{M_d}$$

Coefficient	k_1 $[10^{-2} K/hPa]$	k_2 $[10^{-2} K/hPa]$	k_3 $[10^3 K^2/hPa]$
Smith and Weintraub (1953)	77.61	72.0	375
Boudouris (1963)	77.631	72.006	375.031
Thayer (1974)	77.60	64.80	377.6
Bevis et al. (1994)	77.60	70.4	373.9
Rueger (2002) 'best aver'	77.689	71.295	375.406
Rueger (2002) 'best avail'	77.695	71.97	375.406

Raytracing strategies

	Raytracer	Atmospheric model	Limit [km]	Parameters	Solution
Ground-based					
Hobiger et al. (2008)	KARAT	ECMWF + US 76	86	T, RH: linear P: logarithmically	2D / 3D
Urquhart et al. (2012)	UNB	ECMWF + CIRA 86	100	T, Q: linear P: logarithmically	2D / 3D
Nafisi et al. (2012)	VIE	ECMWF + US 76	76	T: linear P, e: exponential	2D / 3D
Gegout et al. (2014)	GFZ	ECMWF	150	N: log-linear	2D
Zus et al. (2010)	Horizon	ECMWF	80	N: exponential	2D

Summary

1. Raytracing is a complex process.
2. It combines both, meteorological and geodetic reference systems.
3. Can be solved in exact or approximate way.
4. It allows to determine non-traditional parameters:
   - slant and zenith delays,
   - bending angle.

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Thank you for your attention

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